

1) Řešte v R, určete podmínky řešitelnosti

$$\frac{3 - \sin x}{1 - \sin x} = 2$$

$$3 - \sin x = 2 - 2 \sin x$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2} + k2\pi$$

Podmínky:

$$1 - \sin x \neq 0$$

$$\sin x \neq 1 \Rightarrow x \neq \frac{\pi}{2} + k2\pi$$

$$P = \left\{ \frac{3\pi}{2} + k2\pi \right\}, \quad k \in \mathbb{Z}$$

2) Řešte v R, určete podmínky řešitelnosti

$$\cot g\left(\frac{x}{3} + \frac{\pi}{6}\right) = \sqrt{3}$$

$$\frac{x}{3} + \frac{\pi}{6} = a$$

$$\cot g a = \sqrt{3}$$

$$a = \frac{\pi}{6} + k\pi \Rightarrow \frac{x}{3} + \frac{\pi}{6} = \frac{\pi}{6} + k\pi \Rightarrow \frac{x}{3} = k\pi \Rightarrow$$

$$x = 3k\pi$$

Podmínky:

$$\frac{x}{3} + \frac{\pi}{6} \neq k\pi \Rightarrow \frac{x}{3} \neq \frac{11\pi}{6} + k\pi \Rightarrow x \neq \frac{11\pi}{2} + 3k\pi$$

$$P = \{3k\pi\}, \quad k \in \mathbb{Z}$$

3) Řešte v R, určete podmínky řešitelnosti

$$2 \cos^2 x - \sqrt{2} \cos x - 2 = 0$$

$$\cos x = t$$

$$2t^2 - \sqrt{2}t - 2 = 0$$

$$D = 2 + 16 = 18$$

$$t_{1,2} = \frac{\sqrt{2} \pm 3\sqrt{2}}{4} \Rightarrow t_1 = \sqrt{2}, \quad t_2 = -\frac{\sqrt{2}}{2}$$

$$\cos x = \sqrt{2} \Rightarrow P = \emptyset$$

$$\cos x = -\frac{\sqrt{2}}{2} \Rightarrow x_0 = \frac{\pi}{4}, \text{ II., III. kv}$$

$$x = \frac{3\pi}{4} + k2\pi$$

$$x = \frac{5\pi}{4} + k2\pi$$

$$P = \left\{ \frac{3\pi}{4} + k2\pi, \frac{5\pi}{4} + k2\pi \right\}, \quad k \in \mathbb{Z}$$

4) Řešte v \mathbb{R} , určete podmínky řešitelnosti

$$\operatorname{tg} x + \cot x = -2$$

$$\text{vzorec: } \cot x = \frac{1}{\operatorname{tg} x}$$

$$\operatorname{tg} x + \frac{1}{\operatorname{tg} x} = -2$$

$$\operatorname{tg}^2 x + 1 = -2 \operatorname{tg} x$$

$$\operatorname{tg}^2 x + 2 \operatorname{tg} x + 1 = 0$$

$$\operatorname{tg} x = t$$

$$t^2 + 2t + 1 = 0$$

$$(t+1)^2 = 0 \Rightarrow t = -1$$

$$\operatorname{tg} x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi$$

Podmínka:

$$x \neq \frac{\pi}{2} + k\pi \wedge x \neq k\pi$$

$$P = \left\{ \frac{3\pi}{4} + k\pi \right\}, \quad k \in \mathbb{Z}$$