

B

1) Řešte v R, určete podmínky řešitelnosti

$$\frac{3 \operatorname{tg} x - 5}{\operatorname{tg} x - 1} = 4$$

$$3 \operatorname{tg} x - 5 = 4 \operatorname{tg} x - 4$$

$$-\operatorname{tg} x = 1$$

$$\operatorname{tg} x = -1, \quad x_0 = \frac{\pi}{4}, \quad \text{II. kv.}$$

$$x = \frac{3\pi}{4} + k\pi$$

Podmínka:

$$\operatorname{tg} x \neq 1 \Rightarrow x \neq \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}$$

$$P = \left\{ \frac{3\pi}{4} + k\pi \right\}, \quad k \in \mathbb{Z}$$

2) Řešte v R, určete podmínky řešitelnosti

$$\sin\left(3x + \frac{\pi}{3}\right) = -\frac{1}{2}$$

$$3x + \frac{\pi}{3} = a$$

$$\sin a = -\frac{1}{2}$$

$$a_0 = \frac{\pi}{6}, \quad \text{III., IV. kv.}$$

$$a_1 = \frac{7\pi}{6} + k2\pi \Rightarrow 3x + \frac{\pi}{3} = \frac{7\pi}{6} + k2\pi \Rightarrow 3x = \frac{5\pi}{6} + k2\pi \Rightarrow x_1 = \frac{5\pi}{18} + k\frac{2\pi}{3}$$

$$a_2 = \frac{11\pi}{6} + k2\pi \Rightarrow 3x + \frac{\pi}{3} = \frac{11\pi}{6} + k2\pi \Rightarrow 3x = \frac{9\pi}{6} + k2\pi \Rightarrow x_2 = \frac{9\pi}{18} + k\frac{2\pi}{3} = \frac{\pi}{2} + k\frac{2\pi}{3}$$

$$P = \left\{ \frac{5\pi}{18} + k\frac{2\pi}{3}, \quad \frac{\pi}{2} + k\frac{2\pi}{3} \right\}, \quad k \in \mathbb{Z}$$

3) Řešte v \mathbb{R} , určete podmínky řešitelnosti

$$2\cos^2 x + \cos x - 1 = 0$$

$$\cos x = t$$

$$2t^2 + t - 1 = 0$$

$$D = 1 + 8 = 9$$

$$t_{1,2} = \frac{-1 \pm 3}{4} \Rightarrow t_1 = \frac{1}{2}, \quad t_2 = -1$$

$$\cos x = \frac{1}{2} \Rightarrow x_0 = \frac{\pi}{3}, \quad I., IV. kv.$$

$$x_1 = \frac{\pi}{3} + k2\pi$$

$$x_2 = \frac{5\pi}{3} + k2\pi$$

$$P = \left\{ \frac{\pi}{3} + k2\pi, \quad \frac{5\pi}{3} + k2\pi \right\}, \quad k \in \mathbb{Z}$$

4) Řešte v \mathbb{R} , určete podmínky řešitelnosti

Podmínka:

$$x \neq \frac{\pi}{2} + k\pi \wedge x \neq k\pi$$

$$\operatorname{tg} x + \operatorname{cotg} x = 2$$

$$\text{vzorec: } \operatorname{cotg} x = \frac{1}{\operatorname{tg} x}$$

$$\operatorname{tg} x + \frac{1}{\operatorname{tg} x} = 2$$

$$\operatorname{tg}^2 x + 1 = 2 \operatorname{tg} x$$

$$\operatorname{tg}^2 x - 2 \operatorname{tg} x + 1 = 0$$

$$\operatorname{tg} x = t$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0 \Rightarrow t = 1$$

$$\operatorname{tg} x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$$

$$P = \left\{ \frac{\pi}{4} + k\pi \right\}, \quad k \in \mathbb{Z}$$