

# Logaritmické funkce

1) Určete definiční obor funkce

$$f_1: y = \log_5(x+1)$$

$$f_2: y = \log_2 \frac{x+1}{x}$$

$$f_3: y = \log_{10} \frac{1-x}{x+2}$$

$$f_4: y = \sqrt{\log_2 x}$$

$$f_5: y = \sqrt{\log_2 \frac{1}{x}}$$

$$f_6: y = \frac{1}{\sqrt{\log_2 x}}$$

$$f_7: y = \log_3(x^2 - 4x + 4)$$

$$f_8: y = \frac{\log_3(3x-2)}{x^2 - x - 2}$$

$$f_9: y = \sqrt[4]{\log(3x^2 - 2x)}$$

$$f_{10}: y = \sqrt{\log_{\frac{1}{2}} \log_3 \frac{x+1}{x-1}}$$

Pojem: **logaritmus**

Logaritmus je exponent, kterým umocníme základ logaritmu a vyjde argument  $x$

$$a^{\log_a x} = x$$

2) Vypočítejte:

$$\log_7 \sqrt{7} =$$

$$\log_8 \sqrt{2} =$$

$$\log_{\frac{1}{3}} 9 =$$

$$\log_{0,25} 4 =$$

$$\log_5 125 =$$

$$\log_{\sqrt{2}} 16 =$$

$$\log_{\sqrt{5}} 1 =$$

$$\log_{0,2} 0,04 =$$

3) Najděte všechna kladná čísla  $x$ , pro která platí

a)  $\log_3 x = 4$   $x =$

d)  $\log_{\sqrt{2}} x = 4$   $x =$

b)  $\log_{\frac{1}{5}} x = -1$   $x =$

e)  $\log_{\frac{1}{4}} x = \frac{3}{2}$   $x =$

c)  $\log_5 x = 0$   $x =$

f)  $\log_a x = -\frac{3}{5}$   $x =$

4) Najděte všechna kladná čísla  $a$ , pro která platí

a)  $\log_a 27 = 3$   $a =$

d)  $\log_a 4 = \frac{1}{4}$   $a =$

b)  $\log_a \frac{1}{3} = 3$   $a =$

e)  $\log_a \frac{1}{16} = 4$   $a =$

c)  $\log_a 8 = 6$   $a =$

f)  $\log_a \sqrt{8} = 3$   $a =$

5) Doplňte tabulky:

$x$	$\frac{1}{4}$		0		0,5		$-\frac{1}{2}$		$\frac{1}{16}$
$\log_{\frac{1}{2}} x$		-3		3		$-\frac{1}{2}$		0	

$x$	0,01		10		-3		0		$\sqrt[5]{10}$
$\log_{0,1} x$		-2		3		-3		0	

$x$	3		1	
$\log_x x$	$\frac{1}{2}$	-2		1

$x$	0,2	-5		0,04
$\log_x x$	-1		3	

Nakreslete graf logaritmické funkce:

$$f_1 : y = \log_2 x$$

$$f_2 : y = \log_2 (x + 1)$$

$$f_3 : y = \log_2 |x|$$

$$f_4 : y = \log_2 (-x)$$

$$f_5 : y = \log_2 x + 2$$

$$f_6 : y = \log_2 (x - 2) + 3$$

Nakreslete graf logaritmické funkce:

$$f_7 : y = \log_{\frac{1}{2}} (x) - 2$$

$$f_8 : y = \log_{\frac{1}{2}} (x - 2)$$

$$f_9 : y = -\log_{\frac{1}{2}} (x - 2)$$

$$f_{10} : y = \log_{\frac{1}{2}} (x + 1) - 3$$

Rozhodněte o pravdivosti výroků:

$$\log_2 5 > 0$$

$$\log_{\frac{1}{2}} 2 > 0$$

$$\log_2 \frac{1}{2} > 0$$

$$\log_5 2 > 1$$

$$\log_{\frac{1}{3}} 0,2 > 1$$

$$\log_{\frac{1}{3}} \frac{1}{9} > 1$$

$$\log_{\frac{1}{2}} \frac{1}{9} < 1$$

$$\log_{\sqrt{\frac{1}{8}}} \frac{1}{4} < 1$$

$$\log_{\sqrt{0,5}} 5 < 0$$

$$\log_{\sqrt{0,2}} 5 < \log_{\sqrt{0,2}} 2$$

$$\log_{\sqrt{20}} 0,5 < \log_{\sqrt{20}} 2$$

$$\log_{\frac{1}{2}} 3 > \log_{\frac{1}{2}} 2$$

$$\log_{\frac{1}{2}} 2 \leq \log_2 \frac{1}{2}$$

$$\log_a 2 \leq \log_a \frac{1}{2} \Rightarrow a \in (0, 1)$$

$$\log_a \sqrt{3} \leq \log_a 3 \Rightarrow a \in (0, 1)$$

$$\log_a \sqrt{3} \leq \log_a \sqrt{\frac{2}{5}} \Rightarrow a \in (1, \infty)$$

$$\log_a 5,5 \geq \log_a 1,2 \Rightarrow a \in (1, \infty)$$

## Výsledky

1)  $D(f_1) = (-1, \infty)$

$D(f_2) = (-\infty, -1) \cup (0, \infty)$

$D(f_3) = (-2, 1)$

$D(f_4) = (1, \infty)$

$D(f_5) = (0, 1)$

$D(f_6) = (1, \infty)$

$D(f_7) = \mathbb{R} - \{2\}$

$D(f_8) = \left(\frac{2}{3}, 2\right) \cup (2, \infty)$

$D(f_9) = \left(-\infty, \frac{-1}{3}\right) \cup (1, \infty)$

$D(f_{10}) = (2, \infty)$

2)

$\log_7 \sqrt{7} = \frac{1}{2}$

$\log_8 \sqrt[3]{2} = \frac{1}{6}$

$\log_{\frac{1}{3}} 9 = -2$

$\log_{0,25} 4 = -1$

$\log_5 125 = 3$

$\log_{\sqrt{2}} 16 = 8$

$\log_{\sqrt{5}} 1 = 0$

$\log_{0,2} 0,04 = 2$



3) a)  $x = 81$  b)  $x = 5$  c)  $x = 1$  d)  $x = 4$  e)  $x = 0,125$  f)  $x = 10^{-\frac{3}{5}}$

4) a)  $a = 3$  b)  $a = \sqrt[3]{\frac{1}{3}}$  c)  $a = \sqrt{2}$   
d)  $a = 64$  e)  $a = \frac{1}{2}$  f)  $a = \sqrt{2}$

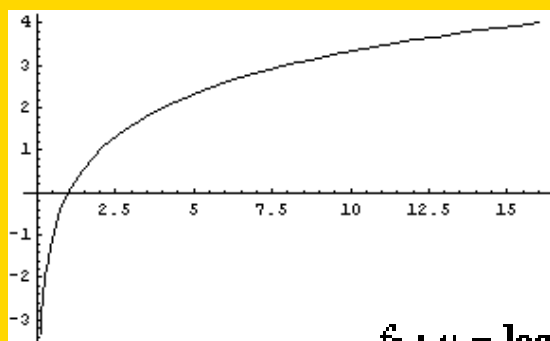
5)

$x$	$\frac{1}{4}$	8	0	$\frac{1}{8}$	0,5	$\sqrt{2}$	$-\frac{1}{2}$	1	$\frac{1}{16}$
$\log_{\frac{1}{2}} x$	2	-3	--	3	1	$-\frac{1}{2}$	--	0	4

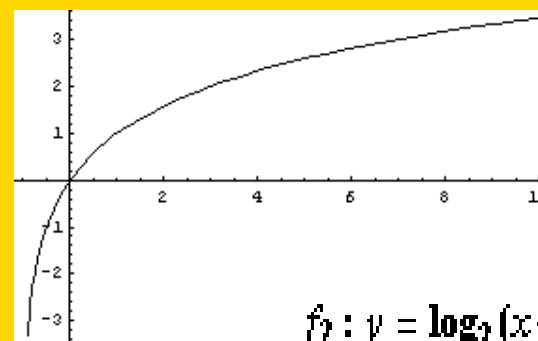
$x$	0,01	100	10	$\frac{1}{1000}$	-3	1000	0	1	$\sqrt[5]{10}$
$\log_{0,1} x$	2	-2	-1	3	--	-3	--	0	$-\frac{1}{5}$

$x$	3	$\frac{1}{81}$	1	9
$\log_3 x$	$\frac{1}{2}$	-2	0	1

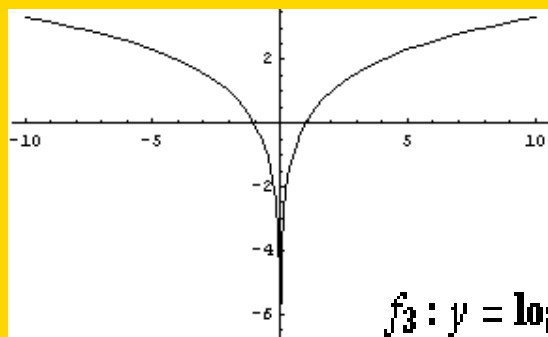
$x$	0,2	-5	125	0,04
$\log_5 x$	-1	--	3	-2



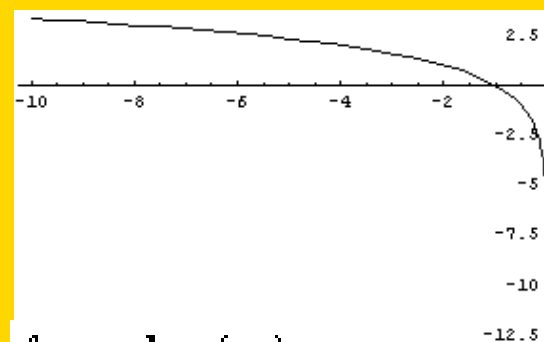
$$f_1: y = \log_2 x$$



$$f_2: y = \log_2(x + 1)$$



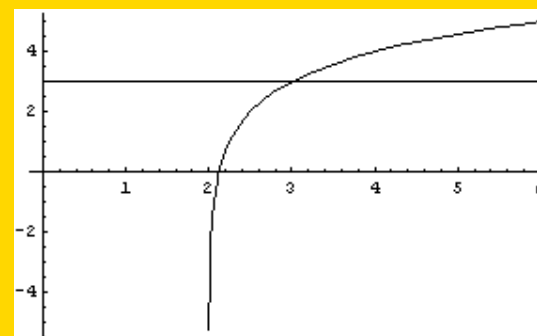
$$f_3: y = \log_2 |x|$$



$$f_4: y = \log_2(-x)$$

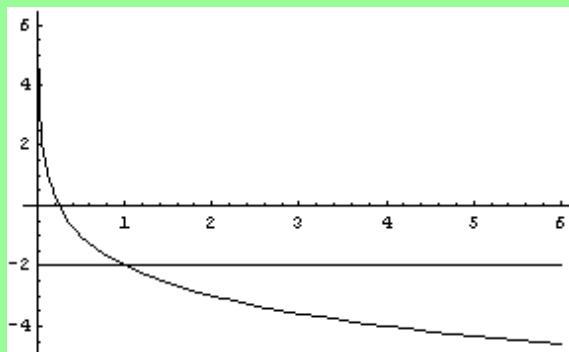


$$f_5: y = \log_2 x + 2$$

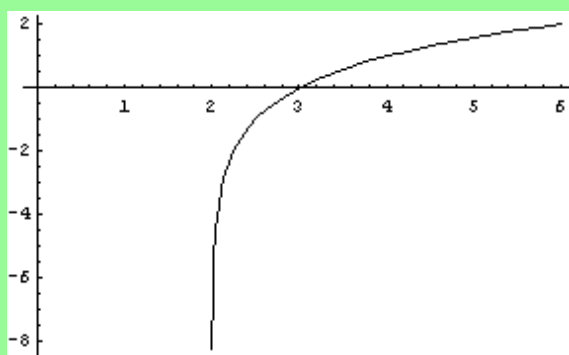
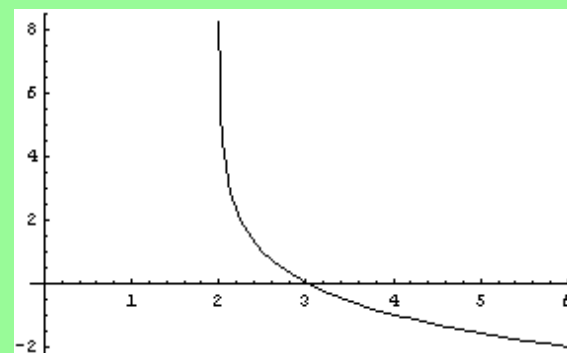


$$f_6: y = \log_2(x - 2) + 3$$

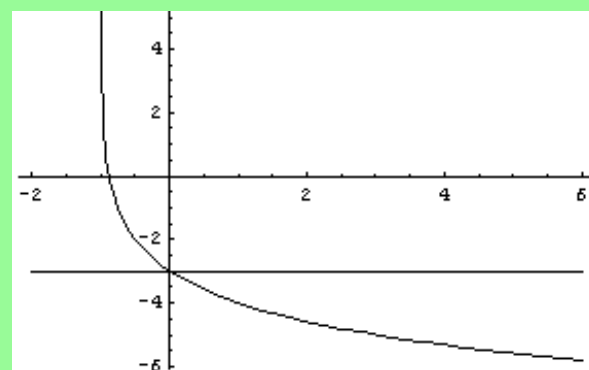
$$f_7: y = \log_{\frac{1}{2}}(x) - 2$$



$$f_8: y = \log_{\frac{1}{2}}(x - 2)$$



$$f_9: y = -\log_{\frac{1}{2}}(x - 2)$$



$$f_{10}: y = \log_{\frac{1}{2}}(x + 1) - 3$$

Rozhodněte o pravdivosti výroků:

$$\log_2 5 > 0$$

pravda

$$\log_{\frac{1}{2}} 2 > 0$$

nepravda

$$\log_2 \frac{1}{2} > 0$$

nepravda

$$\log_5 2 > 1$$

nepravda

$$\log_{\frac{1}{3}} 0,2 > 1$$

pravda

$$\log_{\frac{1}{3}} \frac{1}{9} > 1$$

pravda

$$\log_{81} \frac{1}{9} < 1$$

pravda

$$\log_{\sqrt{8}} \frac{1}{4} < 1$$

pravda

$$\log_{\sqrt{0,5}} 5 < 0$$

pravda

$$\log_{\sqrt{0,2}} 5 < \log_{\sqrt{0,2}} 2$$

pravda

$$\log_{\sqrt{20}} 0,5 < \log_{\sqrt{20}} 2$$

pravda

$$\log_{\frac{1}{2}} 3 > \log_{\frac{1}{2}} 2$$

nepravda

$$\log_{\frac{1}{2}} 2 \leq \log_2 \frac{1}{2}$$

pravda

$$\log_a 2 \leq \log_a \frac{1}{2} \Rightarrow a \in (0, 1)$$

pravda

$$\log_a \sqrt{3} \leq \log_a 3 \Rightarrow a \in (0, 1)$$

nepravda

$$\log_a \sqrt{3} \leq \log_a \sqrt{\frac{2}{5}} \Rightarrow a \in (1, \infty)$$

nepravda

$$\log_a 5,5 \geq \log_a 1,2 \Rightarrow a \in (1, \infty)$$

pravda