

Exponenciální rovnice - rce s neznámou v exponentu

$$2^x + 4^x = 16$$

Způsoby řešení exponenciálních rovnic:

a) $a^x = a^y \Rightarrow x = y$

b) substitucí

c) vytýkáním

d) logaritmicky

Řešené příklady:

$$2^x = 16$$

$$2^x = 2^4$$

$$x = 4$$

$$x = 2$$

$$\text{NSolve}[2^x == 16, x]$$

$$\{x \rightarrow 2.\}$$

$$5^{x-4} = 0,008$$

$$5^{x-4} = \frac{8}{1000}$$

$$5^{x-4} = \frac{1}{125}$$

$$5^{x-4} = 5^{-3}$$

$$x - 4 = -3$$

$$x = 1$$

$$\text{NSolve}[5^{(x-4)} == 0.008, x]$$

$$\{\{x \rightarrow 1.\}\}$$

$$\left(\frac{4}{25}\right)^{x+3} \cdot \left(\frac{125}{8}\right)^{4x-1} = \frac{5}{2}$$

$$\left(\frac{5}{2}\right)^{-2x-6} \cdot \left(\frac{5}{2}\right)^{12x-3} = \frac{5}{2}$$

$$\left(\frac{5}{2}\right)^{10x-9} = \left(\frac{5}{2}\right)$$

$$10x - 9 = 1$$

$$10x = 10$$

$$x = 1$$

$$\text{NSolve}\left[\left(\frac{4}{25}\right)^{x+3} * \left(\frac{125}{8}\right)^{4x-1} == \frac{5}{2}, x\right]$$

$$\{\{x \rightarrow 1.\}\}$$

$$2^{x+7} \sqrt[7]{4^{13-x}} = 1024$$

$$4^{\frac{13-x}{2}} = 2^{10}$$

$$2^{2 \frac{13-x}{2}} = 2^{10}$$

$$\frac{(26 - 2x)}{2x + 7} = 10$$

$$26 - 2x = 20x + 70$$

$$-44 = 22x$$

$$x = -2$$

$$\text{FindRoot}\left[2^{x+7} \sqrt[7]{4^{13-x}} == 1024, \{x, 0\}\right]$$

$$\{x \rightarrow -2.\}$$

Úlohy k řešení:

(kontrolu správnosti proveďte v programu Mathematica)

$$a) \left(1 - \frac{5}{9}\right)^{\frac{2}{3-2x}} = \left(\frac{9}{4}\right)^{\frac{3}{x-3}}$$

$$b) \frac{10^{x^2}}{2^{-15}} = \frac{5^{-15}}{10^{12-12x}}$$

$$c) \left(\frac{4}{9}\right)^x \cdot \left(\frac{27}{8}\right)^{x-1} = \frac{2}{3}$$

$$d) 27^{5x-6} \cdot 81^{2x+3} = 9^{4x-2} \cdot 3^{7x-2}$$

Výsledky:

```
In[86]:= NSolve[ $\left(1 - \frac{5}{9}\right)^{\frac{2}{3-2x}} == \left(\frac{9}{4}\right)^{\frac{3}{x-3}}, x]$ 
```

```
Out[86]= {{x -> 0.75}}
```

```
In[85]:= NSolve[ $\frac{10^{x^2}}{2^{-15}} == \frac{5^{-15}}{10^{12-12x}}, x]$ 
```

```
Out[85]= {{x -> 3.}, {x -> 9.}}
```

```
In[88]:= NSolve[ $\left(\frac{4}{9}\right)^x * \left(\frac{27}{8}\right)^{x-1} == \frac{2}{3}, x]$ 
```

```
Out[88]= {{x -> 2.}}
```

```
In[89]:= NSolve[ $27^{5x-6} * 81^{2x+3} == 9^{4x-2} * 3^{7x-2}, x]$ 
```

```
{x -> 0.}
```

Exponenciální rovnice 2:

$$4^{x+1} - 8 * 4^{x-1} = 32$$

$$4^{x-1} (4^2 - 8) = 32$$

$$4^{x-1} . 8 = 32 / 8$$

$$4^{x-1} = 4$$

$$x - 1 = 1$$

$$x = 2$$

$$\text{NSolve}[4^{x+1} - 8 * 4^{x-1} == 32, x] \quad \{ \{x \rightarrow 2. \} \}$$

JINÝ ZPŮSOB ŘEŠENÍ:

$$4^{x+1} - 8 * 4^{x-1} = 32$$

$$a^{x+y} = a^x + a^y$$

$$a^{x-y} = \frac{a^x}{a^y}$$

$$4 * 4^x - 8 * \frac{4^x}{4} = 32$$

$$4 * 4^x - 2 * 4^x = 32$$

$$2 * 4^x = 32 \quad /: 2$$

$$4^x = 16$$

$$x = 2$$

$$3^{2x-1} + 3^{2x-2} - 3^{2x-4} = 315$$

$$3^{2x-4}(3^3 + 3^2 - 1) = 315$$

$$3^{2x-4} * 35 = 315 \quad /: 35$$

$$3^{2x-4} = 9$$

$$2x - 4 = 2$$

$$x = 3$$

$$\text{NSolve}[3^{2x-1} + 3^{2x-2} - 3^{2x-4} == 315, x]$$

$$\{\{x \rightarrow 3.\}\}$$

Příklady k procvičení:

(zkoušku proveďte výpočtem v programu Mathematica)

a) $5 \cdot 4^{x+1} - 4^{x+2} = 4^{x-1} + 240$

b) $5 \cdot 2^{x+2} - 6 \cdot 3^{x+2} = 3^{x+3} + 2 \cdot 2^{x+1}$

c) $8^{x-1} - 3 = 7 \cdot 8^{x-2} + 5$

d) $5^x + 1 = 3 \cdot 5^{x-1} + 11$

Výsledky úloh

a)

```
In[5]:= 5*4x+1 - 4x+2 == 4x-1 + 240  
NSolve[5*4x+1 - 4x+2 == 4x-1 + 240, x]
```

```
Out[6]= {{x -> 3.}}
```

b)

```
In[7]:= NSolve[5*2x+2 - 6*3x+2 == 3x+3 + 2*2x+1, x]
```

```
Out[7]= {{x -> -4.}}
```

c)

```
In[9]:= NSolve[8x-1 - 3 == 7*8x-2 + 5, x]
```

```
Out[9]= {{x -> 3.}}
```

d)

```
NSolve[5x + 1 == 3*5x-1 + 11, x]
```

```
{{x -> 2.}}
```

Exponenciální rovnice řešené substitucí

$$3^{2x-1} + 3^x - 3^0 = 3^{-1}$$

$$\frac{3^{2x}}{3} + 3^x - 1 = \frac{1}{3} \quad / *3$$

$$3^{2x} + 3 * 3^x - 3 = 1$$

$$3^x = t$$

$$t^2 + 3t - 4 = 0$$

$$t_1 = -4 \quad t_2 = 1$$

$$3^x = -4 \quad \Rightarrow P = \emptyset$$

$$3^x = 1 \quad \Rightarrow 3^x = 3^0 \quad \Rightarrow x = 0$$

$$\text{NSolve}[3^{2x-1} + 3^x - 3^0 == 3^{-1}, x]$$

$$\{x \rightarrow 0.\}$$

$$25^{2x} - 3 * 25^x = 10$$

$$25^{2x} - 3 * 25^x = 10$$

$$25^x = t$$

$$t^2 - 3t - 10 = 0$$

$$t_1 = 5 \quad t_2 = -2$$

$$25^x = 5 \Rightarrow 5^{2x} = 5 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

$$25^x = -2 \Rightarrow P = \emptyset$$

$$\text{NSolve}[25^{2x} - 3 * 25^x == 10, x]$$

$$\{x \rightarrow 0.5\}$$

Řešte v R, zkoušku proved'te v programu Math.

a) $4^x - 10 * 2^{x-1} = 24$

b) $\sqrt[x]{81} + \frac{27}{\sqrt[x]{81}} = 12$

c) $9^{\sqrt{x+2}} = 27 * 3^{\sqrt{x+2}}$

Výsledky:

a) $\text{NSolve}[2^{2x} - 10 * 2^{x-1} == 24, x]$ $\{x \rightarrow 3.\}$

b) $\text{NSolve}\left[\sqrt[x]{81} + \frac{27}{\sqrt[x]{81}} == 12, x\right]$ $\{x \rightarrow 2.\}, \{x \rightarrow 4.\}$

c) $\text{NSolve}[9^{\sqrt{x+2}} == 27 * 3^{\sqrt{x+2}}, x]$ $\{\{x \rightarrow 7.\}\}$

Různé úlohy:

$$2^{3x-1} * 4 = 8^{x+1} * 0.5^x$$

{{x → 2.}}

$$\frac{81}{16} = \left(\frac{2}{3}\right)^x * \left(\frac{9}{4}\right)^{x+1}$$

{{x → 2.}}

$$3^x + 3^{x+1} = 108$$

{{x → 3.}}

$$6 * 7^{x+3} - 7^{x+2} = 41$$

{{x → -2.}}

$$9 * 3^x + 3^{-x} = 10$$

$\{(x \rightarrow -2.), \{x \rightarrow 0.\}\}$

$$16^x = 8 * 4^x + 2 * 8^x$$

$\{x \rightarrow 2.\}$

$$2^{x-1} - 2^{x-2} = 5^{x-3} + 2^{x-3}$$

$\{(x \rightarrow 3.)\}$

$$3^{x+1} + 2 * 3^{-x} = 7$$

$\{(x \rightarrow -1.), \{x \rightarrow 0.63093\}\}$

$$2^x * 3^{x-1} = 12$$

$\{(x \rightarrow 2.)\}$